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Friction Pump Performance Formulation

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ABSTRACT. An attempt was made to formulate the overall performance of the multiple-disc friction pump, first by dimensional analysis, then by deriving and solving the integral momentum equations for laminar flow with respect to a rotating frame attached to the axis of rotation. The performance of the stator is taken into consideration by defining a diffuser conversion efficiency which is then used, in addition to the rotor performance, to describe the overall performance of the pump in terms of the relevant input parameters, and to generate the head and efficiency characteristics. The method was demonstrated for a typical set of values of the governing input parameters.

1. Introduction

Friction pumps are characterized by simple construction and low cost. They have stable, theoretically predictable performance, are highly resistant to cavitation, operate at extremely low noise levels, and can easily be reversed to work as turbines without significant loss of performance. Some of their potential fields of application are handling highly viscous liquids and rarefied gases, as cavitation inhibiting inducers (e.g., for rocket pumps), and as low-noise air conditioning fans.

A typical multiple-disc friction pump is sketched in Fig. 1. Like conventional pumps, it also comprises a rotor assembly and a stator (casing). The rotor carries a large number of closely spaced, corotating discs which are centrally bored to receive the fluid being handled. Each pair of adjacent discs confines one pumping "element". In contradiction to conventional pumps where friction is the cause of energy dissipation and losses, friction pumps depend on viscous friction as the only mechanism to impart angular momentum and transfer energy to the fluid. Therefore, the work transfer process within the rotor of these machines is unavoidably accompanied by dissipation. The radial pressure gradient in the rotor, created by the acquired fluid rotation, tends to drive a through-flow, while viscous shear tends to oppose this through-flow. This fact makes it impossible for a friction pump to attain, even hypothetically, an efficiency of unity.

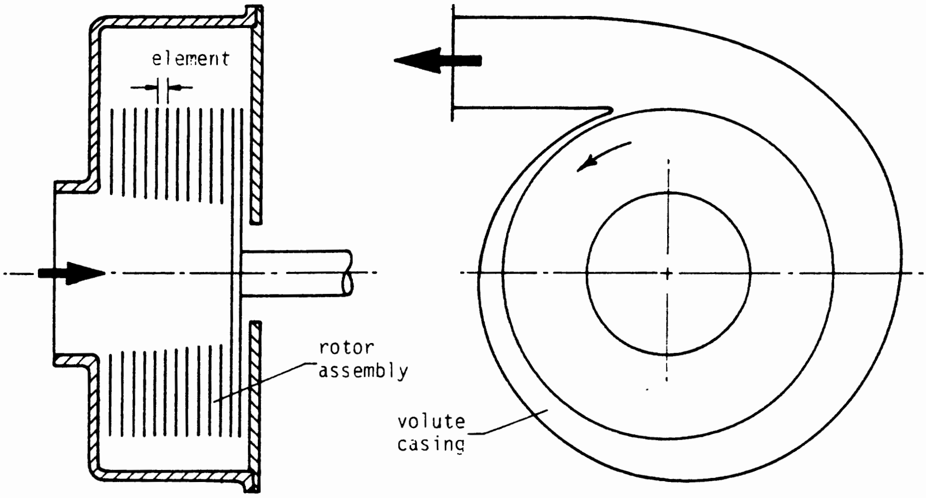

FIG. 1. Schematic of friction pump.

The axial spacing between adjacent discs of a friction pump is very narrow and is of the same order as the disc thickness, Balje[1]. This small gap width is necessary to increase the tangential shear responsible for setting the fluid into rotation. However, a narrow gap increases the resistance to the flow and strongly reduces the generated flow rate. Friction pumps are therefore categorized as lowest specific speed machines and are intended to operate in the lowest machine Reynolds number range. But in this range of low performance, they are superior to centrifugal pumps whose head and efficiency rapidly deteriorate with decreasing Reynolds number, Stepanoff[2] and Piesche[3].

The stator of a friction pump mainly consists of a recuperator following the rotor which eventually includes a vaneless annular diffuser, a scroll casing and a discharge section, and is thus similar to its counterpart in a conventional pump. The main functions of the stator are the guidance of the fluid towards the pump's discharge end, and partial recovery of the high absolute kinetic energy at rotor's exit. Typical literature on friction pumps traditionally concentrates on the flow in the rotor, and almost completely ignores the stator.

In the literature, the flow in the rotor element of a friction pump has been most frequently treated as laminar flow[4-16]. Purely turbulent flow analyses have been undertaken by Murata *et al.*[11] and Piesche[3], and mixed laminar/turbulent flow analysis was conducted by Koehler[17]. In the overwhelming majority of papers, the fluid has been considered as Newtonian and incompressible. Garrison[14], Piesche[15] and Piesche and Felsch[16] addressed compressibility effects, while Mansour[18] recently considered non-Newtonian fluids. Two methods of flow computation are in common use, they are based on either integral or differential formulation; the latter includes the methods of finite differences and finite elements.

In the present work, the governing parameters are first worked out by a simple dimensional analysis. The resulting dimensionless groups are then compared to, and correlated with other forms found in the literature. Thereafter, the integral equations of motion for laminar flow in the rotor are derived directly from the Navier-Stokes equations by partial integration across the axial gap width. In this formulation, a rotating frame is used. The partial integration is made possible by assuming self-similar disc-to-disc parabolic distributions for the radial and tangential components of the relative velocity. These equations are solved numerically, and the solution is used to formulate the performance of the rotor. A global diffuser efficiency is then introduced to describe the performance of the stator, and the overall pump performance is finally obtained by assembling together the performance criteria of the rotor and the stator.

2. Dimensional Analysis

Without loss of generality, the analysis may be limited to just one pumping element of the rotor, *i.e.*, the flow field between two adjacent rotating discs, Fig. 2. Consideration is further limited to fluids with constant properties, thus excluding compressibility and heat transfer phenomenae.

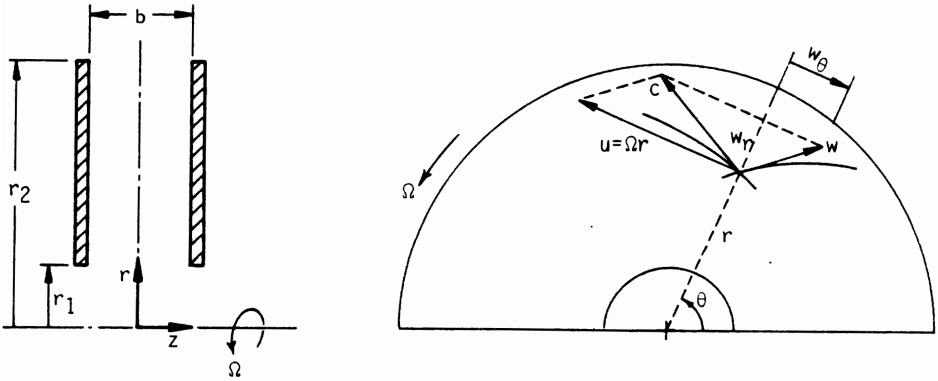

FIG. 2. Pump element and system of coordinates.

The main independent variables governing the rotor performance fall in three categories, *viz.*, geometrical quantities, fluid properties and control variables. Geometrical quantities include inlet radius r_1, outlet radius r_2, and disc spacing b; fluid properties include density ρ and dynamic viscosity μ; and control variables are given by the angular speed of the rotor Ω, the volume flow rate Q and the tangential (swirl) component of the absolute inlet velocity $c_{1\theta}$, if any. As dependent variables, the specific useful energy $E = (p_2 - p_1)/\rho + (c_2^2 - c_1^2)/2 + g(Z_2 - Z_1)$ and the efficiency η are chosen. Frequently, the head H is used in place of E, where $E = gH$; sometimes gH itself is referred to as head. The head and efficiency characteristics (*i.e.*, the dimensional performance laws) may thus be written in the form

$$gH = gH(r_1, r_2, b, \rho, \mu, \omega, Q, c_{1\theta}) \quad (1a)$$

$$\eta = \eta(r_1, r_2, b, \rho, \mu, \omega, Q, c_{1\theta}) \quad (1b)$$

After applying elementary dimensional analysis, the number of independent variables is reduced, and the above performance laws may be rewritten in the following dimensionless forms

$$\psi = \psi(\lambda, \in, Ph, \phi, \gamma) \quad (2a)$$

$$\eta = \eta(\lambda, \in, Ph, \phi, \gamma) \quad (2b)$$

The dimensionless groups appearing in Eqn (2) have the following structures

$$\lambda = \frac{r_1}{b}, \in = \frac{r_2}{r_1}, Ph = b\sqrt{\frac{\Omega}{\nu}}, \phi = \frac{Q}{\Omega r_2^3}, \gamma = \frac{c_{1\theta}}{\Omega r_1}, \Psi = \frac{gH}{\Omega^2 r_2^2} \quad (3)$$

Their meanings are now discussed. Starting with the dependent groups, *i.e.*, the head coefficient $\psi = gH/\Omega^2 r_2^2$ and efficiency η, it is seen that these are defined in a similar way as their counterparts in conventional pumps.

Of the five independent groups on the right side of Eqn (2), the first two, namely $\lambda = r_1/b$ and $\in = r_2/r_1$, are geometrical shape factors; they must have the same values for geometrically similar pumps and need not be considered further if the performance of similar pumps or one particular pump is to be studied.

The third parameter in the list, the so-called Polhausen parameter $Ph = b/\sqrt{\Omega/\nu}$, is a measure of the ratio b/δ between the gap width b and the boundary layer thickness on either disc, δ. Namely, we may recall that on a single rotating disc bounded by an infinitely extending fluid, the boundary layer thickness δ is proportional to $\sqrt{\nu/\Omega}$, Schlichting[19]. The Polhausen parameter has an important influence on the performance, because it determines to what extent the bulk of fluid follows the rotation of the discs, and it controls the shape of the disc-to-disc velocity distribution. Too small Ph values approach the case of solid body rotation, while too large values correspond to almost decoupled boundary layers on the two discs and a nonrotating core in-between. For these reasons, in friction pumps the Polhausen parameter is usually kept within narrow limits, $Ph \leq 2.5-3.5$. The square of Polhausen parameter, $Ph^2 = \Omega b^2/\nu$, is sometimes referred to as a Reynolds number.

The fourth independent parameter $\phi = Q/\Omega r_2^3$ is a flow coefficient, and the fifth $\gamma = c_{1\theta}/\Omega r_1$ is a pre-whirl or swirl coefficient, which normally equals zero, unless inlet guide vanes are fitted before the rotor to create such pre-rotation. A value of $\gamma = 1$ means that the fluid has acquired a tangential velocity component exactly equal to the velocity of the inlet edge of the rotor $c_{1\theta} = \Omega r_1$. Since $\omega_{1\theta} = c_{1\theta} - \Omega r_1$, this would make $\omega_{1\theta} = 0$ and eliminates any tangential shock at entry.

Any other dimensionless parameter of a structure different from the groups listed in Eqn (3) will not be substantially different one, but may be constructed from them by multiplication, raising to a power or a mixed operation. In the literature, dimensionless groups of slightly different structure are frequently encountered. In the following, we quote three further flow coefficient definitions as alternatives to ϕ, and four Reynolds number definitions as alternatives to Ph.

$$\phi_1 = \frac{\bar{c}_{1r}}{\Omega r_1}, \quad \phi_2 = \frac{\bar{c}_{2r}}{\Omega r_2}, \quad \phi_3 = \frac{Q}{\Omega r_1^3}$$

$$Re_1 = \frac{\Omega b^2}{\nu}, \quad Re_2 = \frac{\Omega r_1 b}{\nu}, \quad Re_3 = \frac{\Omega r_1^2}{\nu}, \quad Re_4 = \frac{\Omega r_2^2}{\nu}$$

The quantities $\bar{c}_{1r}$ and $\bar{c}_{1r}$ are the mean values over the gap width of the radial components of the inlet and exit velocities respectively.

It is easy to establish the relations between the above parameters and those in Eqn (3), namely

$$\phi_1 = \frac{1}{2\pi}\lambda \in^3 \phi, \quad \phi_2 = \frac{1}{2\pi}\lambda \in \phi, \quad \phi_3 = \in^3 \phi$$

$$Re_1 = Ph^2, \quad Re_2 = \lambda P h^2, \quad Re_3 = \lambda^2 P h^2, \quad Re_4 = \lambda^2 \in^2 Ph^2$$

It should be noted that if other effects, such as compressibility or variable physical properties ρ, μ, k, ..., are present, other influence parameters like Mach number M, Prandtl number Pr, Eckert number Ec, ... etc, would have to be included.

3. Mathematical Formulation

The describing equations for the flow in the rotor element are formulated with respect to a rotating frame using cylindrical coordinates (r, θ, z), Fig. 2. The relevant velocity in this frame is the relative velocity $\vec{w}$ with components (w_r, w_θ, w_z) in the three principal directions. This relative velocity is related to the absolute velocity $\vec{c}$,seen in a fixed frame by: $\vec{c} = \vec{w} + \vec{\Omega} \times \vec{r} = \vec{w} + \vec{u}$. The two velocities only differ in their tangential components, *i.e.*, $c_r = w_r$, $c_\theta = w_\theta + u = w_\theta + \Omega r$ and $c_z = w_z$.

The following simplifications are introduced: (1) The fluid is Newtonian with constant properties; (2) The flow is steady, $\partial()/\partial t = 0$; (3) The flow is axially symmetric, $\partial()/\partial\theta = 0$; (4) axial velocity component is negligible, $w_z = 0$; (5) The flow is symmetric with respect to the midplane $z = b/2$; (6) Body forces are negligible; (7) Radial gradients are much smaller than axial gradients, $\partial()/\partial r \ll \partial()/\partial z$, $\partial^2()/\partial r^2 \ll \partial^2()/\partial z^2$.

The symmetry with respect to the midplane $z = b/2$ makes it possible to limit consideration to the half region ($0 \le z \le b/2$). Subject to the above conditions, the Navier-Stokes and continuity equations, and the associated boundary conditions may be written as follows:

3.1 Momentum Equations

$$w_r \frac{\partial w_r}{\partial r} - \left(\Omega^2 r + 2\Omega w_\theta + \frac{w_\theta^2}{r} \right) + w_z \frac{\partial w_r}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu \frac{\partial^2 w_r}{\partial z^2} \tag{4}$$

$$w_r \frac{\partial w_\theta}{\partial r} + \frac{w_r w_\theta}{r} + 2\Omega w_r + w_z \frac{\partial w_\theta}{\partial z} = \nu \frac{\partial^2 w_\theta}{\partial z^2} \tag{5}$$

$$\frac{\partial p}{\partial z} = 0 \tag{6}$$

3.2 Continuity Equation

$$\frac{\partial w_r}{\partial r} + \frac{w_r}{r} + \frac{\partial w_z}{\partial z} = 0 \tag{7}$$

3.3 Boundary Conditions

$$\begin{aligned} r = r_1: &\quad w_r = \overline{w}_{1r}, \quad w_\theta = c_{1\theta} - u \\ z = 0, b: &\quad w_r = 0, \quad w_\theta = 0 \\ z = \frac{b}{2}: &\quad \frac{\partial w_r}{\partial z} = \frac{\partial w_\theta}{\partial z} = 0 \end{aligned} \tag{8}$$

The degenerating Eqn (6) implies $p \neq p(z)$, and the axial symmetry condition $\partial/\partial\theta = 0$ implies $p \neq p(\theta)$. Therefore, the pressure only varies in radial direction, *i.e.*, $p = p(r)$, and $\partial p/\partial r = dp/dr$.

4. Solution

A crucial step in all integral methods of solution is the suggestion of meaningful velocity profiles. In the present case, these profiles are $w_r(r, z)$ and $w_\theta(r, z)$ since w_z is assumed to be absent. Rice and co-workers[7-10] developed an integral method in which polynomial velocity profiles $\Sigma_{i=0}^{N} a_i(r) z^i$ with $N \geq 4$ are used. Their formulation is capable of handling the $3D$ flow in the entrance region where $w_z \neq 0$, and can accomodate large gap widths. The simpler approaches of Hasinger and Kehrt[5], and Nendl[6] are based on parabolic disc-to-disc radial and tangential velocity distributions which implicitely neglect the entrance length. Namely, in analogy to pipe flow, we can anticipate that the entrance length is proportional to the gap width b and depends on the Reynolds number *Re* (or the Polhausen parameter *Ph*). Since these both are small quantities in friction pumps, e.g., $Ph \leq 2.5$-3.5, the entrance length is expected to be small, and the assumption appears to be justified. For the sake of simplicity, this approach is adopted in the present work. The two non-vanishing velocity components $w_r(r, z)$ and $w_\theta(r, z)$ are then expressed in a product form as follows

$$w_r(r,z) = \overline{w}_{1r} \cdot q(r) h(z) \tag{9a}$$

$$w_\theta(r,z) = \overline{w}_{1\theta} \cdot f(r) s(z) \tag{9b}$$

where the bar above a quantity indicates its mean value over the gap width at any radial station.

$$q(r) = \frac{\overline{w}_r(r)}{w_{1r}}, \qquad f(r) = \frac{\overline{w}_\theta(r)}{w_{1\theta}}, \qquad h(z) = \frac{w_r(r,z)}{w_r(r)}, \qquad s(z) = \frac{w_\theta(r,z)}{w_\theta(r)}$$

The h and s functions are arbitrarily taken to be parabolic as follows, c.f.[6]:

$$h(z) = s(z) = 6\frac{z}{b}\left(1 - \frac{z}{b}\right) \tag{10}$$

The integrated form of the continuity Eqn (7) gives immediately the solution for the q-function as

$$q(r) = \frac{r_1}{r} \tag{11}$$

If the momentum equations, Eqn (4) and (5), are then partially integrated with respect to z over the integration domain $(0, b/2)$, the following ordinary differential equations are obtained

$$\frac{dP}{dR} = R + 2(\gamma - 1)f + k(\gamma - 1)^2 \frac{f^2}{R} + k\phi_1^2 \frac{1}{R^3} - \frac{12\phi_1}{Ph^2}\frac{1}{R} \tag{12}$$

$$\frac{df}{dR} = -\left(\frac{1}{R} + \frac{12}{k}\frac{1}{Ph^2\phi_1}R\right)f - \frac{2}{k(\gamma - 1)} \tag{13}$$

where dimensionless notation has been introduced according to the following definitions

$$R = \frac{r}{r_1}, \qquad P = \frac{p - p_1}{\rho \Omega^2 r_1^2} \qquad k = 2\int_0^{1/2} h(Z)s(Z)dz \qquad \text{and} \qquad Z = \frac{z}{b}$$

The approach followed in the present work in deriving Eqn (12) and (13) directly from the Navier-Stokes and continuity equations is different from that followed by Nendl[6]. The initial conditions are given by

$$R = 1: \quad f = 1, \quad P = 0 \tag{14}$$

Equations (12) and (13) form a set of two coupled, first order ordinary differential equations. Together with the initial conditions, Eqn (14), they constitute an initial-value problem. This problem is well suited to numerical solution by a standard Runge-Kutta routine that advances the solution step by step from the known inlet section ($R = 1$) towards the exit section ($R = \epsilon$). Once $f(R)$ and $P(R)$ have been solved for, the performance can be easily calculated.

5. Performance

Most authors confine their analysis to the flow in the rotor only. Such an approach exaggerates both head and efficiency of the pump. Namely, the high kinetic head at rotor's exit is counted in such formulations as useful head, but in fact it cannot be completely recovered as static head in the following recuperator passages, especially in this low Reynolds number flow situation. A primary design goal would be the recovery of the largest possible part of this kinetic head by careful design of the diffuser passages. The success of such an effort can be described by a suitably defined conversion efficiency. In the following, a diffuser efficiency of the recuperator, η_d, is defined and included in the overall performance evaluation.

5.1 Heads

The theoretical head, expressed as gH_{th}, is defined as the one which would be obtained if the work done by the rotor on the fluid were fully available without any dissipation; it is directly obtained by applying Euler's equation of turbomachinery[1]

$$g H_{th} = u_2 c_{2\theta} - u_1 c_{1\theta} \tag{15a}$$

Applying this to the present case we obtain

$$g H_{th} = u_2^2 \left[1 + \frac{k}{\epsilon} f(\epsilon)(\gamma - 1) - \frac{\gamma}{\epsilon^2} \right] \tag{15b}$$

or in dimensionless form, with $\psi_{th} = g H_{th} / \Omega^2 r_2^2$

$$\psi_{th} = 1 + \frac{k}{\epsilon} f(\epsilon)(\gamma - 1) - \frac{\gamma}{\epsilon^2} \tag{16}$$

However, this theoretical head coefficient is not completely available at rotor's exit, because part of it is lost within the rotor by different dissipation mechanisms. Rotor losses are attributed to viscous shear in both tangential and radial directions, in addition to tangential entrance shock (if $\gamma \neq 1$).

The total head coefficient at rotor's exit, $\psi_{0,rotor}$, is composed of a static and a dynamic part, *viz.*,

$$\psi_{0,rotor} = \psi_{s,rotor} + \psi_{d,rotor} \tag{17}$$

These head coefficients are obtainable from the solution of Eqn (12)-(14), namely

$$\psi_{s,rotor} = \frac{p_2 - p_1}{\rho u_2^2} = \frac{P(\epsilon)}{\epsilon^2} \tag{18}$$

$$\psi_{d,rotor} = \frac{(\bar{c}_2^2 - c_1^2)/2}{u_2^2}$$

$$= \frac{1}{2}\left[\left(1 + \frac{\gamma}{\epsilon}(\gamma - 1) f(\epsilon) \right)^2 - \frac{\gamma^2}{\epsilon^2} - \frac{\phi_1^2}{\epsilon^2}\left(1 - \frac{1}{\epsilon^2} \right) \right] \tag{19}$$

Since at pump's discharge the velocity is of the same order as at pump's intake, the high velocity at rotor's exit will have to be reduced in the following stator passages. The static head recovery in the recuperator can be expressed as

$$\psi_{s,recup} = \eta_d \psi_{d,rotor} \tag{20}$$

and the pump's useful head becomes

$$\psi = \psi_{s,rotor} + \psi_{s,recup} = \psi_{s,rotor} + \eta_d \psi_{d,rotor} \tag{21}$$

5.2 Reaction

A commonly used definition of the degree of reaction that is valid both for pumps and compressors is the ratio of static enthalpy rise in the rotor to static enthalpy rise in the whole pump. In the present work an alternative definition that is frequently used for pumps and fans, c.f., Pfleiderer[20] and Eck[21], is adopted. In this definition static enthalpies are replaced by static pressures

$$^\circ R = \frac{\psi_{s,rotor}}{\psi_{0,rotor}} \tag{22}$$

It is worth noting that both definitions of reaction become identical for incompressible fluids undergoing isentropic processes because for such processes changes in enthalphy and changes in pressure are directly proportional, as can be seen from Gibbs equation, $Tds = dh - dp/\rho$.

5.3 Efficiencies

The rotor's efficiency is defined as

$$\eta_{rotor} = \frac{\psi_{0,rotor}}{\psi_{th}} \tag{23}$$

The hydraulic efficiency which expresses the hydraulic performance of the whole pump, including the stator is given by

$$\eta_h = \frac{\psi}{\psi_{th}} \tag{24}$$

and the overall efficiency is obtained by multiplying with the mechanical efficiency

$$\eta = \eta_m \eta_h \tag{25}$$

The mechanical efficiency is defined as the rotor gross power mgH_{th} divided by the input shaft power $mgH_{th} + \Delta W_m$, where ΔW_m is the mechanical power losses external to the rotor, e.g., in bearings, seals and disc friction, *i.e.*,

$$\eta_m = \frac{mgH_{th}}{mgH_{th} + \Delta W_m} \tag{26a}$$

Defining a power loss coefficient

$$\xi_m = \frac{\Delta W_m}{2\pi\rho\Omega^3 br_2^4}$$

Eqn (26a) may be rewritten in a dimensionless form as

$$\eta_m = \approx \frac{\phi\psi_{th}}{\phi\psi_{th} + \xi_m} = \frac{1}{1 + \xi_m/(\phi\psi_{th})} \tag{26b}$$

Following Pfleiderer[20] and Eck[21], it may be assumed that ΔW_m is almost exclusively governed by the speed of rotation. Consequently, along a constant-speed characteristic both ΔW_m and ζ_m may be regarded as constant.

6. Discussion and Conclusion

The solution of the describing system of equations, Eqn [12]-[14], and the performance evaluation with the help of Eqn [16]-[25] has been programmed for any given input. This input consists of the radius ratio $\in$, the Polhausen parameter *Ph*, the prewhirl factor γ, the diffuser efficiency η_d and the mechanical power loss coefficient ζ_m. As an example, the representative set of input values ($\in = 2.5$, $Ph = 3$, $\gamma = 0$, $\eta_d = 0.5$, $\zeta_m = 0.05$) is assigned. The performance quantities ψ, $°R$, η_h and η are then computed for various values of the flow coefficient $\phi_2 = \bar{c}_{2r}/\Omega r_2$. All head coefficients and the degree of reaction are plotted versus ϕ_2 in Fig. 3, while efficiencies are plotted separately in Fig. 4.

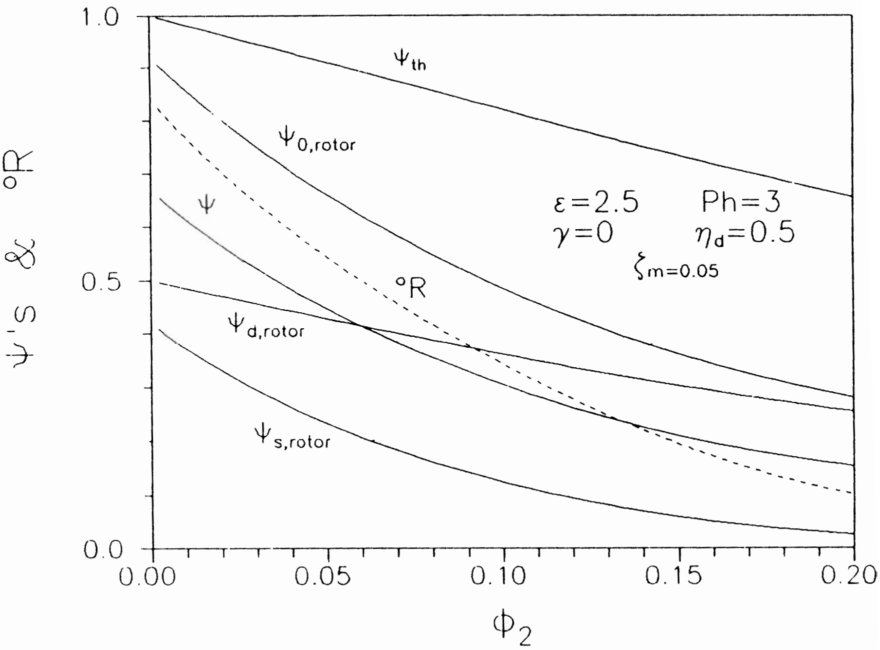

FIG. 3. Head characteristics and reaction.

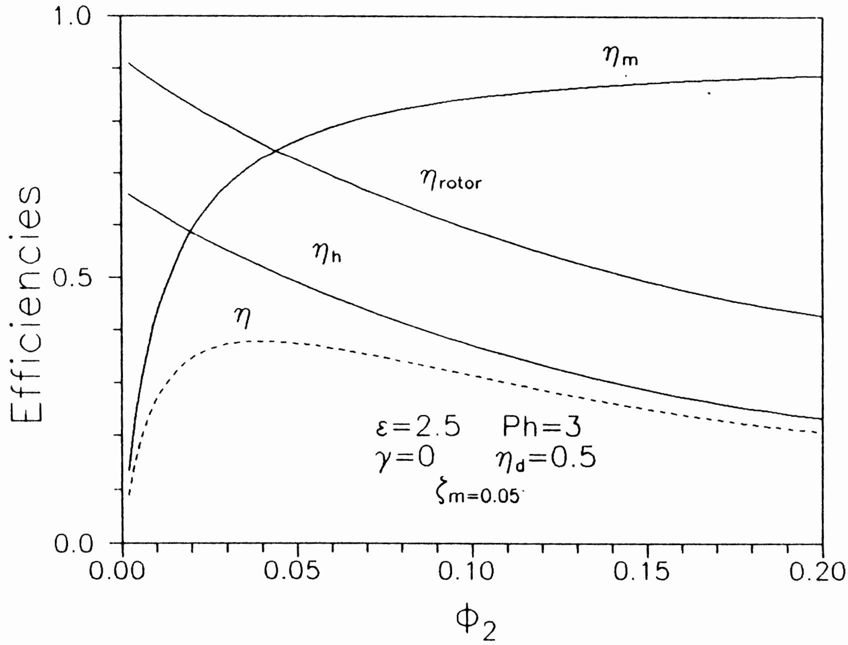

FIG. 4. Efficiency characteristics.

From Fig. 3 it is seen that all head coefficients and the degree of reaction increase monotonically with decreasing flow coefficient. This confirms the fact that friction pumps are better performers at lower flow rates than they are at higher flow rates. As seen from Fig. 4, the same trends are true for the rotor efficiency and the hydraulic efficiency. However, this monotonous behaviour is not maintained for the overall efficiency. The overall efficiency goes through a maximum before dropping back to zero at shutoff. The optimum flow coefficient at which this maximum occurs depends on the value of the Polhausen parameter, being lower for higher *Ph* values.

In concluding this brief discussion, it may be noted that although the above curves were produced for laminar flow using a simple integral approach based on parabolic velocity distributions, the trends depicted remain valid for more sophisticated integral or differential flow methods.

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Nomenclature

b	gap per element
c	absolute velocity
f	$\overline{w}_\theta / \overline{w}_{1\theta}$, function
g	gravity constant; function
h	function
H	head
k	shape factor
$\dot{m}$	mass flow rate
p	pressure
P	$P - P_1 / \rho u_1^2$
Ph	Polhausen parameter
q	function
Q	volume flow rate
r	radial coordinate
s	function
R	r / r_1
°R	degree of reaction
Re	Reynolds number
t	time
u	Ωr
w	relative velocity
W	power (rate of work done)
z	axial coordinate
Z	z/b, also elevation above datum

Greek Symbols

γ	prewhirl factor
ϵ	r_2/r_1
η	efficiency
λ	r_1 / b
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
ϕ	flow coefficient
ψ	head coefficient
Ω	angular speed of rotor
ζ	power loss coefficient

Subscripts

o	total
1	at inner radius
2	at outer radius
d	dynamic, diffuser
h	hydraulic
m	mechanical
r	radial
s	static
z	axial
θ	tangential
1θ	tangential component at inlet

صياغة أداء مضخة احتكاك

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المستخلص . يعتبر هذا البحث محاولة لصياغة الأداء الإجمالي لمضخة احتكاك متعددة الأقراص ، أولاً بإجراء تحليل أبعاد ، وثانيًا باشتقاق معادلات الكم الحركي التكاملية للسريان الطبقي منسوبة إلى نظام محاور إحداثية تدور حول محور الدوران ، ثم حل هذه المعادلات . وقد تم أخذ أداء العضو الساكن في الاعتبار عن طريق تعريف كفاءة تحويل خاصة به ، باعتباره ناشرًا ، ثم استعمال هذه الكفاءة إلى جانب معايير أداء الدوران لإيجاد الأداء الإجمالي للمضخة بدلالة البرامترات المؤثرة ، ومن ثم توليد معادلات خصائص العلو والكفاءة . وقد تم استعراض تطبيق هذه الطريقة بإعطاء قيم نمطية لمجموعة البرامترات المؤثرة (المدخلات) .