JKAU: Eng. Sci., Vol. 2, pp. 133-151 (1410 A.H. / 1990 A.D.)

Design of Rectangular Beams under Torsion, Bending and Shear

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ABSTRACT. A set of interactive strength equations based on the skew-bending model have been modified for designing rectangular beams subjected to torsion, bending and shear. The design procedure is a trial and error approach, and is based on the estimation of the required pure flexural moment capacity of the section to be designed. Torsion-bending-shear interaction data of a reference section have been used in estimating the pure flexural moment capacity. These data are presented in a tabular form and also as non-dimensional interaction diagrams. Four numerical examples covering different possible modes of failure are presented. The torsional moment capacities of the designed sections are compared with those given by the ACI code torsion equations.

Introduction

Reinforced concrete beams under uneven floor loading as in the case of an edge beam in a building, are subjected to torsional moments. The fact that this affects the structural performance of members was long recognized by designers as well as by the ACI code[1]. Lack of adequate research, however, hindered the formulation of any suitable code provisions for the design of reinforced concrete members subjected to torsion. Subsequently, the ACI code[2] included design provisions, on the basis of Hsu's[3] work, for members subjected to torsion and torsion with shear. These, however, neglect the influence of bending moment on the torsional strength of beams, although the interdependence of torsional moment and bending moment capacities of reinforced concrete members was long indicated by different researchers[4-6]. This interdependence becomes more pronounced at low ranges of T/M ratios.

Since the late sixties, researchers have been investigating the torsion-bending interaction[7-11] as well as the torsion-bending-shear interaction[12-14]. Their works are based either on the skew-bending model developed by Lessig[5] or on the space struss analogy propounded by Rausch[15]. Most of these works, however, concentrate on the analytical rather than the design aspect of the torsion problem.

Recently, Hasnat and Akhtaruzzaman[16] presented theoretical equations based on the skew-bending model for the entire range of torsion-bending-shear interaction for rectangular beams of solid cross-section as well as for beams containing a small opening. The equations are suitable for analyzing a given section and can be used to obtain its theoretical torsional moment capacity T_n, under any combination of torsion, bending and shear. In their present format, however, the equations are not suitable for application to design problems. In this paper, the basic strength equations are presented in a different form rendering them applicable to the design of rectangular beams under any loading combination.

Basic Equations

The skew-bending model categorizes torsional failure of a reinforced concrete beam under three different modes: Mode 1, Mode 2 and Mode 3 depending on the location of a skewed compression zone near the top, side or bottom of the section, respectively. The failure pattern depends on the aspect ratio of the beam, the ratio between top and bottom longitudinal reinforcements, and the ratio between applied torsional moment and bending moment in combination with different values of shear force.

According to Hasnat and Akhtaruzzaman[16], T_1, T_2 and T_3, the torsional moment capacities in Modes 1, 2 and 3, respectively, of a rectangular beam of solid section are given by

$$T_1 = \frac{2M_{01}\,K_1}{\Delta}\left[\sqrt{\frac{1}{K_1} + \frac{1}{(\psi\Delta)^2}} - \frac{1}{\psi\Delta}\right] \tag{1}$$

$$T_2 = \frac{2\,M_{01}}{1 + \delta}\sqrt{R_2\,K_2} \tag{2}$$

$$T_3 = \frac{2M_{01}\,K_1}{\Delta'}\left[\frac{1}{\psi\Delta'} - \sqrt{\frac{R_3}{K_1} + \frac{1}{(\psi\Delta')^2}}\right] \tag{3}$$

where

$$K_1 = \frac{(1 + 3\alpha)\,r}{(1 + 2\alpha)^2} \tag{4}$$

$$K_2 = \frac{(3 + \alpha)\,\alpha r}{(2 + \alpha)^2} \tag{5}$$

$$R_2 = \frac{M_{02}}{M_{01}} \tag{6}$$

$$R_3 = \frac{M_{03}}{M_{01}} \tag{7}$$

$$r = \frac{A_w f_{wy}}{s} \frac{0.9\, x_1\, y_1}{M_{01}} \tag{8}$$

The smallest of the three torsional moment capacities, T_1, T_2 and T_3, is taken as the theoretical torsional moment capacity T_n, of the section.

In the above expressions A_w is the area of one leg of vertical stirrups, f_{wy} is the yield strength of stirrup steel, M_{01}, M_{02} and M_{03} represent the bending moment capacities of the section in positive, lateral and negative bending, respectively, s is the stirrup spacing, x_1 and y_1 are the center-to-center dimensions of a long stirrup, α is the aspect ratio of the beam section, δ is a factor incorporating torsional moment and shear force acting at the section, Δ and Δ' are factors incorporating torsional moment, bending moment and shear force acting at the section and its cross sectional dimensions, and ψ is the ratio between torsional moment and bending moment acting at the section.

Transformation of the Basic Equations

Equations 1, 2 and 3 indicate that the values of T_1, T_2 and T_3 can be readily obtained once the various sectional and loading parameters are known.

In a design problem, however, where the sectional parameters are to be established, the equations cannot be readily used. Therefore, Eqs. 1, 2 and 3 are to be expressed in different forms to render them suitable for design applications. The required transformations of the basic equations are presented below.

Transformation of Eq. 1

By transposing and squaring both sides, Eq. 1 is written as

$$\left[\frac{T_1\, \Delta}{2M_{01}\, K_1} + \frac{1}{\psi\Delta}\right]^2 = \frac{1}{K_1} + \frac{1}{(\psi\Delta)^2} \tag{9}$$

Expanding the terms within the parentheses and cancelling $1/(\psi\Delta)^2$ from both sides,

$$\frac{T_1^2\, \Delta^2}{4M_{01}^2\, K_1^2} + \frac{T_1}{M_{01}\, K_1 \psi} = \frac{1}{K_1} \tag{10}$$

or,

$$K_1 = \frac{T_1^2\, \Delta^2}{4M_{01}^2} \cdot \frac{M_{01}\, \psi}{M_{01}\, \psi - T_1} \tag{11}$$

$$= \frac{k_1^2\, \Delta^2}{4} \cdot \frac{\psi}{\psi - k_1} \tag{12}$$

where $$k_1 = \frac{T_1}{M_{01}} \tag{13}$$

Substituting for T_1 from Eq. 1,

$$k_1 = \frac{2K_1}{\Delta}\left[\sqrt{\frac{1}{K_1} + \frac{1}{(\psi\Delta)^2}} - \frac{1}{\psi\Delta}\right] \tag{14}$$

Equation 12 is the transformed form of Eq. 1.

Transformation of Eq. 2

By transposing and squaring both sides, Eq. 2 is written as

$$\frac{T_2^2(1+\delta)^2}{4M_{01}^2} = R_2 K_2 \tag{15}$$

or $$K_2 = \frac{T_2^2(1+\delta)^2}{4M_{01}^2 R_2} \tag{16}$$

$$= \frac{k_2^2(1+\delta)^2}{4R_2} \tag{17}$$

where $$k_2 = \frac{T_2}{M_{01}} \tag{18}$$

Substituting for T_2 from Eq. 2,

$$k_2 = \frac{2}{1+\delta}\sqrt{R_2 K_2} \tag{19}$$

Replacing T_2 by T_n ($= T_u/\phi$) for Mode 2 failure, where T_u and ϕ are the factored torsional moment and the undercapacity factor, respectively, Eq. 18 becomes

$$M_{01} \doteq \frac{M_n \psi}{k_2} \tag{20}$$

where $$\psi = \frac{T_n (= T_2)}{M_n} \tag{21}$$

Equation 17 is the transformed form of Eq. 2.

Transformation of Eq. 3

By transposing and squaring both sides, Eq. 3 is written as

$$\left[\frac{T_3\,\Delta'}{2M_{01}\,K_1} - \frac{1}{\psi\Delta'} \right]^2 = \frac{R_3}{K_1} + \frac{1}{(\psi\Delta')^2} \tag{22}$$

Expanding the terms within the parentheses and cancelling $1/(\psi\Delta')^2$ from both sides,

$$\frac{T_3^2\,\Delta'^2}{4M_{01}^2\,K_1^2} - \frac{T_3}{M_{01}\,K_1\,\psi} = \frac{R_3}{K_1} \tag{23}$$

or

$$K_1 = \frac{T_3^2\,\Delta'^2}{4M_{01}^2} \left[\frac{M_{01}\,\psi}{R_3\,M_{01}\,\psi + T_3} \right] \tag{24}$$

$$= \frac{k_3^2\,\Delta'^2\,\psi}{4\,(R_3\,\psi + k_3)} \tag{25}$$

where

$$k_3 = \frac{T_3}{M_{01}} \tag{26}$$

Substituting for T_3 from Eq. 3,

$$k_3 = \frac{2K_1}{\Delta'} \left[\frac{1}{\psi\Delta'} - \sqrt{\frac{R_3}{K_1} + \frac{1}{(\psi\Delta')^2}} \right] \tag{27}$$

Equation 25 is the transformed form of Eq. 3 and is similar to that of Eq 1.

Equations 12, 17 and 25 are used for simultaneous application of torsion, bending and shear, can be readily used for torsion and bending only by putting $\Delta = 1$, $\delta = 0$ and $\Delta' = -1$. These substitutions reduce Eqs. 12, 17 and 25 to

$$K_1 = \frac{k_1^2}{4} \cdot \frac{\psi}{\psi - k_1} \tag{28}$$

$$K_2 = \frac{k_2^2}{4R_2} \tag{29}$$

$$K_1 = \frac{k_3^2\,\psi}{4(R_3\,\psi + k_3)} \tag{30}$$

Design Procedures

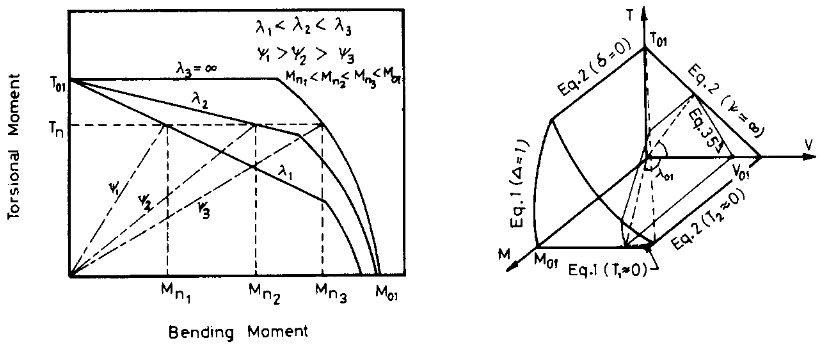
The design for torsion basically means determining the size and spacing of stirrups *required to develop* a desired torsional moment capacity T_n ($= T_u/\phi$) in a section when it is subjected to a factored bending moment M_u and a factored shear force V_u.

The section can be designed first for flexure to have a pure bending moment capacity M_{01} and then the required stirrup size and spacing can be determined to give adequate shear and torsional strengths. The pure bending moment capacity M_{01}, however, is not known initially and as shown in Fig. 1(a), depending on ψ and λ, the ratio between bending moment and shear force acting on the section, the factored moment M_u ($= \phi M_n$) is only a variable fraction of M_{01}. The pure flexural moment capacity M_{01}, nevertheless, can be obtained by using Eq. 13, 18 or 26 written as

$$M_{01} = \frac{T_1}{k_1} \tag{31}$$

$$M_{01} = \frac{T_2}{k_2} \tag{32}$$

$$M_{01} = \frac{T_3}{k_3} \tag{33}$$



(a) Effect of ψ and λ on M_n at constant T_n

(b) Torsion-Bending-Shear interaction surface

FIG. 1. Typical torsion-bending interaction diagrams and torsion-bending-shear interaction surface.

If k_1, k_2 and k_3 are known, the pure flexural moment capacity M_{01} can be obtained by using the least of these with T_1, T_2 or T_3 replaced by T_n in the corresponding equation. However, the values of k_1, k_2 and k_3 themselves depend, among other things, on the section parameters, and so cannot be known initially. This problem can be overcome by a trial and error approach using a reference section of known dimensions and preferably having other variables identical to the section to be designed. The k_1, k_2 and k_3 values of the reference section can be used to obtain an estimated pure flexural moment capacity M'_{01}.

A set of values of k_1 and k_2 of a typical 250 × 500mm reference section are presented in Table 1. The section has 10mm diameter stirrups at 100mm spacing with two 12mm diameter hangers, and concrete strength f'_c of 27.6 MPa, f_{wy} and f_y, the yield strengths of stirrup and longitudinal steel, of 276 MPa.

Table 1 contains values of k_1 and k_2 for three different steel ratios, and a range of T/M ratio (ψ) and M/V ratio (λ). For other values of these parameters, k_1 and k_2 can be interpolated. The table does not contain any values of k_3. The possibility of Mode 3 failure is checked separately as will be shown later. Although Table 1 is for particular values of material strengths and sectional properties, it can be used in any initial trial calculation, for other values of these also. The subsequent calculations are self-adjusting in nature.

TABLE 1. Values of k_1 and k_2 of a 250 × 500mm reference section.

Ratio T/M	$\rho = \rho_{max} = 0.0370$							$\rho = 0.50\,\rho_{max} = 0.0185$							$\rho = 0.25\,\rho_{max} = 0.00925$						
	at λ = 0.05 m	at λ = 0.10 m	at λ = 0.20 m	at λ = 0.60 m	at λ = 1.00 m	at λ = 1.50 m	at λ = ∞	at λ = 0.05 m	at λ = 0.10 m	at λ = 0.20 m	at λ = 0.60 m	at λ = 1.00 m	at λ = 1.50 m	at λ = ∞	at λ = 0.05 m	at λ = 0.10 m	at λ = 0.20 m	at λ = 0.60 m	at λ = 1.00 m	at λ = 1.50 m	at λ = ∞
0.02	0.0012	0.0024	0.0047	0.0132	0.0166	0.0179	0.0198	0.0016	0.0031	0.0062	0.0159	0.0178	0.0187	0.0199	0.0022	0.0044	0.0087	0.0174	0.0187	0.0193	0.0199
0.04	0.0024	0.0047	0.0091	0.0243	0.0313	0.0338	0.0384	0.0031	0.0062	0.0119	0.0304	0.0341	0.0360	0.0391	0.0044	0.0087	0.0169	0.0337	0.0364	0.0377	0.0395
0.06	0.0035	0.0069	0.0132	0.0338	0.0441	0.0476	0.0551	0.0047	0.0091	0.0174	0.0435	0.0489	0.0517	0.0570	0.0066	0.0129	0.0247	0.0489	0.0530	0.0550	0.0583
0.08	0.0049	0.0091	0.0171	0.0419	0.0553	0.0597	0.0696	0.0062	0.0119	0.0225	0.0553 0.0551	0.0622	0.0659	0.0734	0.0087	0.0169	0.0320	0.0630	0.0685	0.0713	0.0762
0.10	0.0058	0.0122	0.0208	0.0490	0.0650	0.0701	0.0819	0.0076	0.0147	0.0274	0.0644	0.0741	0.0786	0.0881	0.0108	0.0209	0.0389	0.0761	0.0828	0.0863	0.0929
0.12	0.0069	0.0132	0.0243	0.0552	0.0736 0.0739	0.0792	0.0924	0.0091	0.0174	0.0320	0.0726	0.0849	0.0900	0.1011	0.0129	0.0247	0.0454	0.0882	0.0961	0.1003	0.1085
0.14	0.0080	0.0152	0.0276	0.0607	0.0798	0.0871	0.1013	0.0105	0.0200	0.0363	0.0798	0.0946	0.1002	0.1127	0.0149	0.0284	0.0516	0.0995	0.1084	0.1130	0.1228
0.16	0.0091	0.0171	0.0308	0.0658	0.0847	0.0941	0.1088	0.0119	0.0225	0.0405	0.0863	0.1033	0.1093	0.1229	0.0169	0.0320	0.0575	0.1099	0.1197	0.1249	0.1358
0.18	0.0101	0.0190	0.0338	0.0700	0.0891	0.1003	0.1154	0.0133	0.0250	0.0444	0.0921	0.1113	0.1176	0.1319	0.0189	0.0355	0.0630	0.1197	0.1301	0.1358	0.1479
0.20	0.0112	0.0208	0.0366	0.0739	0.0929	0.1057 0.1066	0.1210	0.0147	0.0274	0.0481	0.0973	0.1185	0.1250	0.1399	0.0209	0.0389	0.0683	0.1287	0.1398	0.1458	0.1588
0.22	0.0122	0.0226	0.0393	0.0775	0.0963	0.1094	0.1259	0.0161	0.0297	0.0517	0.1020	0.1250	0.1317	0.1470	0.0228	0.0422	0.0734	0.1371	0.1487	0.1550	0.1688
0.24	0.0132	0.0243	0.0419	0.0808	0.0993	0.1121	0.1302	0.0174	0.0320	0.0551	0.1063	0.1310 0.1306	0.1378	0.1533	0.0247	0.0454	0.0782	0.1449	0.1569	0.1636	0.1780
0.26	0.0142	0.0260	0.0443	0.0838	0.1019	0.1143	0.1340	0.0187	0.0342	0.0583	0.1103	0.1341	0.1433	0.1590	0.0266	0.0485	0.0828	0.1522	0.1646	0.1714	0.1864
0.28	0.0152	0.0276	0.0467	0.0866	0.1044	0.1163	0.1374	0.0200	0.0363	0.0614	0.1139	0.1373	0.1484	0.1641	0.0284	0.0516	0.0872	0.1590	0.1717	0.1787	0.1940
0.30	0.0162	0.0292	0.0490	0.0891	0.1066	0.1181	0.1404	0.0213	0.0384	0.0644	0.1172	0.1402	0.1530	0.1687	0.0302	0.0546	0.0914	0.1655	0.1784	0.1854	0.2011
0.32	0.0171	0.0308	0.0511	0.0914	0.1086	0.1198	0.1431	0.0225	0.0405	0.0673	0.1203	0.1428	0.1573 0.1476	0.1729	0.0320	0.0575	0.0954	0.1715 0.1707	0.1846	0.1917	0.2076
0.34	0.0181	0.0323	0.0532	0.0936	0.1104	0.1212	0.1456	0.0238	0.0425	0.0699	0.1231	0.1452	0.1595	0.1767	0.0337	0.0603	0.0993	0.1747	0.1903	0.1976	0.2135
0.36	0.0190	0.0338	0.0552	0.0956	0.1121	0.1226	0.1478	0.0250	0.0444	0.0726	0.1258	0.1474	0.1613	0.1802	0.0355	0.0630	0.1030	0.1786	0.19.57	0.2030	0.2191
0.38	0.0199	0.0352	0.0571	0.0975	0.1136	0.1238	0.1498	0.0262	0.0463	0.0751	0.1283	0.1494	0.1629	0.1834	0.0372	0.0657	0.1066	0.1821	0.2008	0.2081	0.2242
0.040	0.0208	0.0366	0.0589	0.0993	0.1150	0.1249	0.1517 0.1509	0.0274	0.0481	0.0775	0.1306	0.1513	0.1644	0.1864	0.0389	0.0683	0.1100	0.1854	0.2056	0.2129	0.2290
0.42	0.0217	0.0380	0.0607	0.1009	0.1163	0.1259	0.1509	0.0286	0.0499	0.0798	0.1327	0.1530	0.1657	0.1891	0.0405	0.0709	0.1133	0.1884	0.2101	0.2174	0.2334
0.44	0.0226	0.0393	0.0624	0.1024	0.1176	0.1269	0.1509	0.0297	0.0517	0.0820	0.1348	0.1546	0.1670	0.1916	0.0422	0.0734	0.1165	0.1913	0.2143	0.2216	0.2375
0.46	0.0234	0.0406	0.0640	0.1039	0.1187	0.1278	0.1509	0.0309	0.0534	0.0842	0.1369	0.1562	0.1681	0.1940	0.0438	0.0758	0.1195	0.1940	0.2183	0.2255	0.2413
0.48	0.0243	0.0419	0.0656	0.1053	0.1198	0.1287	0.1509	0.0320	0.0551	0.0863	0.1385	0.1576	0.1692	0.1961	0.0454	0.0782	0.1224	0.1966	0.2220	0.2292	0.2450
0.50	0.0252	0.0431	0.0671	0.1066	0.1208	0.1294	0.1509	0.0331	0.0567	0.0883	0.1402	0.1589	0.1702	0.0982 0.1986	0.0470	0.0805	0.1253	0.1990	0.2256 0.2255	0.2327	0.2484
0.60	0.0292	0.0489	0.0739	0.1121	0.1249	0.1325	0.1509	0.0384	0.0644	0.0973	0.1474	0.1644	0.1744	0.1986	0.0546	0.0914	0.1381	0.2092	0.2333	0.2477 0.2475	0.2626
0.70	0.0330	0.0542	0.0798	0.1163	0.1281	0.1349	0.1509	0.0434	0.0713	0.1049	0.1530	0.1685	0.1775	0.1986	0.0617	0.1012	0.1489	0.2172	0.2392	0.2519	0.2734
0.80	0.0366	0.0589	0.0847	0.1198	0.1306	0.1367	0.1509	0.0481	0.0775	0.1115	0.1576	0.1718	0.1799	0.1986	0.0683	0.1100	0.1583	0.2237	0.2438	0.2553	0.2818 0.2819
0.90	0.0399	0.0632	0.0891	0.1226	0.1325	0.1382	0.1509	0.0526	0.0831	0.1172	0.1613	0.1744	0.1818	0.1986	0.0746	0.1180	0.1664	0.2289	0.2475	0.2580	0.2819
1.00	0.0431	0.0671	0.0929	0.1249	0.1342	0.1393	0.1509	0.0567	0.0883	0.1222	0.1644	0.1765	0.1833	0.1986	0.0805	0.1253	0.1735	0.2333	0.2506	0.2602	0.2819
1.50	0.0566	0.0823	0.1066	0.1325	0.1393	0.1430	0.1509	0.0745	0.1083	0.1402	0.1744	0.1833	0.1881	0.1986	0.1057	0.1538	0.1990	0.2475	0.2602	0.2671	0.2819
2.00	0.0671	0.0929	0.1150	0.1367	0.1421	0.1449	0.1509	0.0883	0.1222	0.1513	0.1799	0.1869	0.1906	0.1986	0.1253	0.1735	0.2148	0.2553	0.2653	0.2706	0.2819
2.50	0.0755	0.1006	0.1208	0.1393	0.1438	0.1461	0.1509	0.0993	0.1324	0.1589	0.1833	0.1891	0.1922	0.1986	0.1409	0.1879	0.2255	0.2602	0.2685	0.2728	0.2819
3.00	0.0823	0.1066	0.1249	0.1412	0.1449	0.1469	0.1509	0.1083	0.1402	0.1644	0.1857	0.1906	0.1932	0.1986	0.1538	0.1990	0.2333	0.2636	0.2706	0.2743	0.2819
4.00	0.0929	0.1150	0.1306	0.1435	0.1464	0.1479	0.1509	0.1222	0.1513	0.1718	0.1888	0.1926	0.1945	0.1986	0.1735	0.2148	0.2438	0.2679	0.2734	0.2761	0.2819
5.00	0.1006	0.1208	0.1342	0.1449	0.1473	0.1485	0.1509	0.1324	0.1589	0.1765	0.1906	0.1937	0.1953	0.1986	0.1879	0.2255	0.2506	0.2706	0.2750	0.2773	0.2819
6.00	0.1066	0.1249	0.1367	0.1459	0.1479	0.1489	0.1509	0.1402	0.1644	0.1799	0.1919	0.1945	0.1959	0.1986	0.1990	0.2333	0.2553	0.2724	0.2761	0.2780	0.2819
8.00	0.1150	0.1306	0.1400	0.1471	0.1486	0.1494	0.1509	0.1513	0.1718	0.1842	0.1935	0.1955	0.1965	0.1986	0.2148	0.2438	0.2615	0.2747	0.2775	0.2790	0.2819
10.0	0.1208	0.1342	0.1421	0.1479	0.1491	0.1497	0.1509	0.1589	0.1765	0.1869	0.1945	0.1961	0.1969	0.1986	0.2255	0.2506	0.2653	0.2761	0.2784	0.2796	0.2819
15.0	0.1294	0.1393	0.1449	0.1489	0.1497	0.1501	0.1509	0.1702	0.1833	0.1906	0.1959	0.1969	0.1975	0.1986	0.2416	0.2602	0.2706	0.2780	0.2796	0.2803	0.2819
20.0	0.1342	0.1421	0.1464	0.1494	0.1500	0.1503	0.1509	0.1765	0.1869	0.1926	0.1965	0.1974	0.1978	0.1986	0.2506	0.2653	0.2734	0.2790	0.2801	0.2807	0.2819
25.0	0.1372	0.1438	0.1473	0.1497	0.1502	0.1504	0.1509	0.1805	0.1891	0.1937	0.1969	0.1976	0.1979	0.1986	0.2563	0.2685	0.2750	0.2796	0.2805	0.2810	0.2819
30.0	0.1393	0.1449	0.1479	0.1499	0.1503	0.1505	0.1509	0.1833	0.1906	0.1945	0.0972	0.1978	0.1980	0.1986	0.2602	0.2706	0.2761	0.2799	0.2807	0.2811	0.2819
40.0	0.1421	0.1464	0.1486	0.1502	0.1505	0.1506	0.1509	0.1869	0.1926	0.1955	0.1976	0.1980	0.1982	0.1986	0.2653	0.2734	0.2776	0.2804	0.2810	0.2813	0.2819
50.0	0.1438	0.1473	0.1491	0.1503	0.1506	0.1507	0.1509	0.1891	0.1937	0.1961	0.1978	0.1981	0.1983	0.1986	0.2685	0.2750	0.2784	0.2807	0.2812	0.2814	0.2819
∞	0.1509	0.1509	0.1509	0.1509	0.15097	0.1509	0.1509	0.1509	0.1986	0.1986	0.1986	0.1986	0.1986	0.1986	0.1986	0.2819	0.2819	0.2819	0.2819	0.2819	0.2819

The k_1 values are enclosed by thick lines.

In Table 1, the k_1 values are boxed separately. At any ψ, only the controlling value of k_1 or k_2, *i.e.*, the smaller of the two, is listed. The ψ at which both values are listed is a transitional ψ between Mode 1 and Mode 2 failures, or is very close to it.

The values of k_1 and k_2 can also be presented as a set of non-dimensional torsion-bending interaction diagrams. A typical set of such diagrams for the 250 × 500mm reference section with $\rho = 0.50\ \rho_{max}$ are shown in Fig. 2. The ordinate of any of the interaction diagrams corresponding to a given combination of ψ and λ, gives the value of k_1 or k_2 depending on the controlling mode of failure indicated in the figure.

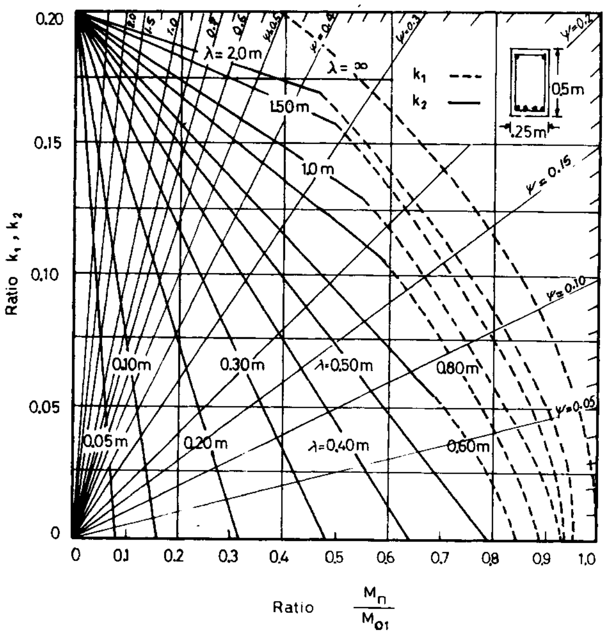


FIG. 2. Interaction diagrams for the 250 × 500mm reference section.

Once M'_{01} is estimated, as described above, the section is then designed for flexure. Next, depending on whether k_1 or k_2 was used to find M'_{01}, the appropriate equations are used to carry out the torsion design of the section.

In the next section, four design examples are presented; Examples 1 and 2 show designs of Mode 1 and Mode 2 controlled cases, respectively, while Example 3 shows a case with its design ψ very close to a transitional ψ of the reference section. Example 4 illustrates how the torsional moment capacity in Mode 3 is enhanced, if required, as discussed below.

After a section is designed as described, its torsional capacities in Modes 1, 2 and 3 are checked. Since only k_1 or k_2 was used in obtaining M'_{01}, the checking may indicate a premature failure in Mode 3, especially if the design ψ is high or the section has a low top steel ratio, ρ'. In that case, the design is modified by increasing the top steel area A'_s. Equation 23 can be expressed in a different form to obtain the value of R_3 required to ensure adequate strength in Mode 3. Replacing T_3 by T_n for Mode 3 failure and cancelling $1/K_1$ from both sides, Eq. 23 can be written as

$$R_3 = \frac{T_n^2 \, \Delta'^2}{4M_{01}^2 \, K} - \frac{T_n}{M_{01} \, \psi} \tag{34}$$

Equation 34 gives the required value of R_3. Next, the corresponding A_{03} and hence the top steel area A'_s required to give an adequate torsional capacity in Mode 3 can be obtained to finalize the design as will be illustrated in Example 4 as mentioned earlier.

After a section has been designed for flexure and torsion as outlined above, its shear capacity is checked. This is of particular importance in cases where $\lambda_{01}(= M_{01}/V_{01}$ where V_{01} is the pure shear capacity λ) is greater than or close to the design λ. This is because, as indicated in Fig. 1(b), at this range of λ, the shear capacity of a section is overestimated by Eq. 2.

Collins *et al.*[8] suggested that the reduced nominal shear capacity $\bar{V}_n$, of a section subjected to torsion, bending and shear, can be obtained from

$$V_{01} = \bar{V}_n + 1.6 \frac{T_n}{b} \tag{35}$$

in which the pure shear capacity V_{01}, is given by

$$V_{01} = V_c + V_s \tag{36}$$

$$= 0.17 \sqrt{f'_c} \, bd + \frac{2 A_w f_{wy} d}{s} \tag{37}$$

where b and d represent the width and effective depth of the section, respectively.

If $\bar{V}_n \geq V_u/\phi$, then the shear capacity of the section is deemed to be adequate. However, if $\bar{V}_n < V_u/\phi$, then the shear capacity of the section is to be increased as illustrated in Examples 1 and 2.

The design procedures discussed above are listed below.

A. Design for flexure

1. k_1 or k_2 is obtained from Table 1 for the design values of ψ and λ, and an assumed value of ρ.

2. Estimated pure flexural moment M'_{01}, is obtained at $T_u/\phi(k_1$ or $k_2)$ and a section is designed for flexure using M'_{01}. Then steps (3) through (5) are followed if

Mode 1 controls (for $k_1 < k_2$). Otherwise, steps (6) through (8) are followed.

B. Design for torsion (Mode 1 controls)

3. Values of M_{01}, α, μ, Δ and $k_1(= T_u/\phi\ M_{01})$ of the section designed in step (2) are obtained, where μ is a factor involving section dimensions.

4. Equations 12 and 8 are used to find K_1 and r in terms of stirrup spacing s for an assumed size of stirrup.

5. The values of K_1 and r are substituted in Eq. 4 to obtain stirrup spacing s. The calculations may be repeated for a different stirrup size until a suitable spacing is obtained. A practical spacing is selected and step (9) is carried out.

C. Design for torsion (Mode 2 controls)

6. Values of M_{02}, R_2 and δ of the section designed in step (2) are obtained.

7. Equations 16 and 8 are used to find K_2 and r in terms of s for an assumed size of stirrup.

8. The values of K_2 and r are substituted in Eq. 5 to find stirrup spacing s. The calculations are repeated until a suitable spacing is obtained. A practical spacing is selected and step (9) is carried out.

D. Checking shear capacity

9. Equations 37 and 35 are used to get V_{01} and $\bar{V}_n$, respectively. If $\bar{V}_n \geq V_u/\phi$, step (11) is performed. Otherwise, step (10) is carried out.

10. $\bar{V}_n$ is replaced by V_u/ϕ in Eq. 35 to obtain the required enhanced V_{01} which is then substituted in Eq. 37 to obtain the stirrup spacing needed to satisfy the shear requirement. A practical spacing is selected and step (11) is performed.

E. Checking torsion capacity

11. The torsional moment capacities in Modes 1, 2 and 3, of the section are checked by using Eqs. 1, 2 and 3 to ensure $T_n \leq T_1$, T_2 and T_3.

12. If $T_3 < T_n$, Eq. 34 is used to ensure adequate torsional moment capacity in Mode 3 by providing the required top steel area.

Design Examples

Four design examples covering Modes 1, 2 and 3 as well as shear governed failure cases are presented. Some of these have α and ρ different from those of the 250 × 500mm reference section illustrating the usefulness of Table 1 for such cases. Examples 1, 2 and 3 have been designed with $\rho = 0.5\ \rho_{max}$ (=0.0185) and Example 4 with ρ = 0.0120. The nominal values of torsional moment, bending moment and shear force used in the examples, are presented in Table 2. In all the examples, f'_c is taken as 27.6 MPa with f_{wy} and f_y as 276 MPa. Results of calculations are presented in Tables 4, 5 and 6. Figure 3 shows the sections selected at various stages of design.

TABLE 2. Data used in the design examples.

Example No. (1)	$\frac{T_u}{0.85}$ (kN-m) (2)	$\frac{M_u}{0.90}$ (kN-m) (3)	$\frac{V_u}{0.85}$ (kN) (4)	$\psi = \frac{\text{col. (2)}}{\text{col. (3)}}$ (5)	$\lambda = \frac{\text{col. (3)}}{\text{col. (4)}}$ (m) (6)	Assumed steel ratio (7)	From Table 1 k_1 (8)	From Table 1 k_2 (9)	$M'_{01} = \frac{\text{col. (2)}}{\text{col. (8) or (9)}}$ (kN-m) (10)
1	10.0	250.0	420.0	0.04	0.60	0.0185*	0.0304	-	328.95
2	75.0	150.0	250.0	0.50	0.60	0.0185*	-	0.1402	534.95
3	115.0	360.0	240.0	0.32	1.50	0.0185*	0.1573	0.1576	731.09
4	120.0	30.0	150.0	4.00	0.20	0.0120	-	0.2224	539.57

*$\rho = 0.5\,\rho_{max}$

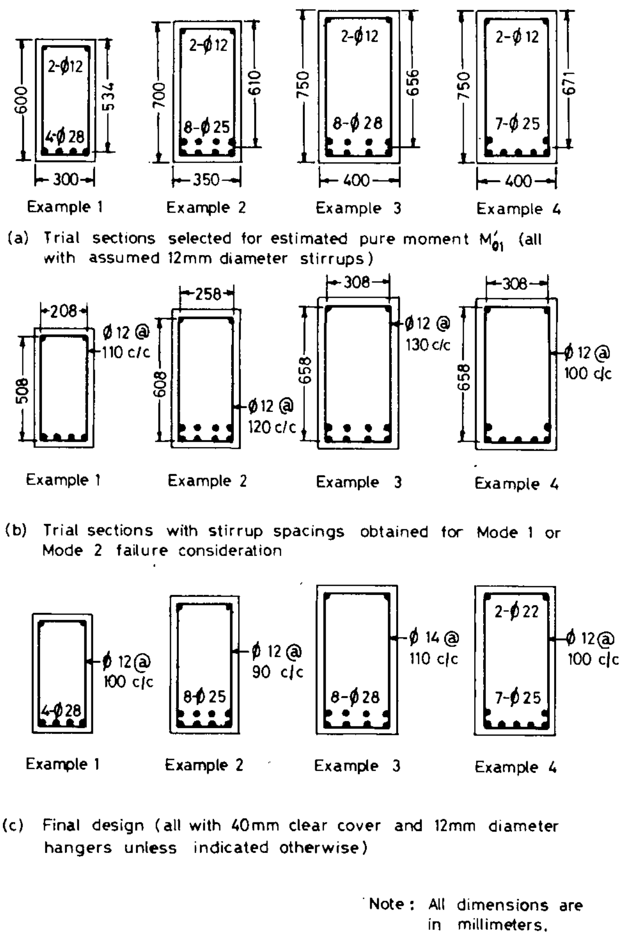


FIG. 3. Details of beam sections designed.

Example 1

Design a rectangular beam section if T_u = 8.5 kN-m, M_u = 225.0 kN-m and V_n = 357.0 kN.

Design for flexure

The design values of ψ and λ along with that of k_1 obtained from Table 1, are presented in Table 2. It also contains the estimated required pure flexural moment M'_{01} (= 328.95 kN-m). Preliminary calculations indicate that a 300 × 600mm section with four 28mm diameter bars will be satisfactory. The pure flexural moment capacity M_{01} of the section is computed, with assumed 10mm diameter stirrups and 40mm clear cover, as 331.16 kN-m.

The α, μ, Δ and Δ' values of the trial section are identical to those listed in Table 4.

Design for torsion

Since Mode 1 behavior is indicated, replacing T_1 by T_u/ϕ in Eq. 13, k_1 = 0.03020. Substituting the values of k_1, ψ and Δ in Eq. 12, K_1 = 0.02128. Also, substituting the value of α (= 2.0) in Eq. 4, K_1 = 0.280r. Equating the two values of K_1, r = 0.0760.

Next, from Eq. 8, for 10mm diameter stirrups, r = 6.306/s. Thus, s = 82.9mm. Therefore, 10mm diameter stirrups may be used at 80mm spacing. However, this spacing may be deemed to be too small and a larger size, *i.e.*, 12mm diameter stirrup can be used as shown herein.

The trial section with 12mm diameter stirrups is shown in Fig. 3(a), and its details are listed in Table 3. The flexural moment capacities of the section in positive, lateral

TABLE 3. Details of the trial sections shown in Fig. 3(a)

Example No. (1)	b (mm) (2)	h (mm) (3)	A_s (mm²) (4)	d (mm) (5)	d_2 (mm) (6)	d_3 (mm) (7)	x_1 (mm) (8)	y_1 (mm) (9)
1	300	600	2460	534	254	542	208	508
2	350	700	3930	610	304	642	258	608
3	400	750	4930	656	354	692	308	658
4	400	750	3440	671	354	692	308	658

TABLE 4. Flexural moment capacities and other parameters of the sections shown in Fig. 3(a).

Example No. (1)	M_{01} (kN-m) (2)	M_{02} (kN-m) (3)	M_{03} kN-m) (4)	$R_2 = \frac{M_{02}}{M_{01}}$ (5)	$R_3 = \frac{M_{03}}{M_{01}}$ (6)	$\alpha = \frac{b}{h}$ (7)	$\mu = \frac{b^2 + bh}{2b + 4h}$ (mm) (8)	$\delta = \frac{b}{2\psi\lambda}$ (9)	$\Delta = 1 + \frac{\mu}{\psi\lambda}$ (10)	$\Delta' = -1 + \frac{\mu}{\psi\lambda}$ (11)	$r = \frac{A_w f_{wy}\, 0.9\, x_1 y_1}{s\ M_{01}}$ (12)
1	329.80	89.28	33.53	0.27070	0.10166	2.000	90.0	6.3025	4.782	2.782	8.993/s
2	589.76	164.26	39.82	0.27852	0.06752	2.000	105.0	0.5833	1.350	−0.650	7.446/s
3	793.69	237.43	42.97	0.29915	0.05414	1.875	121.1	0.4167	1.252	−0.748	7.170/s
4	588.57	171.69	42.97	0.29171	0.07301	1.875	121.1	0.2500	1.150	−0.850	9.665/s

and negative bending are given in Table 4. Using M_{01} = 329.80 kN-m from Table 4 and replacing T_1 by T_u/ϕ in Eq. 13, k_1 = 0.03032. Substituting the values of k_1, ψ and Δ in Eq. 12, K_1 = 0.02172. Equating this with the value of K_1 given by Eq. 4, r = 0.07757.

Also, as shown in Table 4, Eq. 8 gives $r = 8.993/s$. Equating the two values of r, s = 115.9mm. Therefore, 12mm diameter stirrups are selected with 110mm spacing as shown in Fig. 3(b).

Checking for shear

The pure shear capacity V_{01}, and λ_{01} of the section shown in Fig. 3(b) are computed in Table 5. From Eq. 35, $\bar{V}_n$ = 392.6 kN < V_u/ϕ (= 420.0 kN).

TABLE 5. Pure shear capacity and λ_{01} of the sections shown in Fig. 3(b).

Example No.	b	d	$V_c = 0.17\sqrt{f'_c}\,bd$	s	$V_s = \frac{2A_w f_{wy} d}{s}$	$V_{01} = V_c + V_s$	M_{01}	$\lambda_{01} = \frac{M_{01}}{V_{01}}$
	(mm)	(mm)	(kN)	(mm)	(kN)	(kN)	(kN-m)	(m)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	300	534	143.1	110	302.8	445.9	329.81	0.740
2	350	610	190.7	120	317.1	507.8	589.76	1.160
3	400	656	234.4	130	314.7	549.1	793.69	1.445
4	400	671	239.7	100	418.5	658.2	588.57	0.894

The required enhanced pure shear capacity of the section is, therefore, obtained by replacing $\bar{V}_n$ by V_u/ϕ in Eq. 35 as $420 + 1.6T_u/\phi b$ = 473.3 kN. Next, putting V_{01} = 473.3 kN in Eq. 37, s = 100.9mm. Therefore, the stirrup spacing is reduced from 110mm to 100mm as shown in Fig. 3(c).

Torsional moment capacity

The torsional moment capacities in Modes 1, 2 and 3 of the section shown in Fig. 3(c), are computed in Table 6. The computed values of r, R_2, R_3, K_1 and K_2 of the section are also listed in the table.

TABLE 6. Calculation of torsional moment capacities of the sections shown in Fig. 3(c).

Example No.	r from Eq. (8)	K_1 from Eq. (4)	k_1 from Eq. (14)	$T_1 = k_1 M_{01}$ (kN-m)	R_2 from Eq. (6)	K_2 from Eq. (5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	0.08993	0.02518	0.0312	10.29	0.27070	0.05621
2	0.08295	0.02323	0.1805	106.45	0.27852	0.05184
3	0.08826	0.02592	0.1740	137.63	0.29925	0.05373
4	0.09665	0.02838	0.2822	166.10	0.33218	0.05883

Table 6 (contd)

k_2 from Eq. (19) (8)	$T_2 = k_2 M_{01}$ (kN-m) (9)	R_3 from Eq. (7) (10)	k_3 from Eq. (27) (11)	$T_3 = k_3 M_{01}$ (kN-m) (12)	T_n (kN-m) (13)	Expected failure mode (14)
0.0338	11.15	0.10169	–0.0040	– 1.32	10.29	1
0.1518	89.53	0.06752	0.4714	278.01	89.53	2
0.1790	141.58	0.05416	0.5928	468.92	137.63	1
0.2236	131.60	0.21297	0.2154	126.80	126.78	3

Table 6 shows that T_1, T_2 and T_3 are 10.29, 11.15 and –1.32 kN-m, respectively. The negative sign of T_3 indicates that Mode 3 failure is not feasible. Thus, the theoretical torsional moment capacity T_n, is 10.29 kN-m and the failure will be in Mode 1.

Example 2

Design a rectangular beam section if T_u = 63.75 kN-m, M_u = 135.0 kN-m and V_u = 212.5 kN.

The estimated flexural moment capacity M'_{01}, is given in Table 2. A corresponding preliminary trial section is shown in Fig. 3(a). Various details of the section are presented in Tables 3 and 4. Since Mode 2 behavior is indicated, Eq. 16 is used to get K_2 = 0.03639. Thus, from Eq. 5 and 8, s = 128.2mm. A spacing of 120mm is, therefore, selected as shown in Fig. 3(b).

Table 5 shows the pure shear capacity V_{01} and λ_{01} of the section. Substituting the value of V_{01} in Eq. 35, $\bar{V}_n$ = 164.9 kN $< V_u/\phi$. Thus, as in Example 1, the pure shear capacity of the section is increased to 250.0 + 1.6 $T_u/\phi b$ = 592.9 kN.

Substituting V_{01} = 592.9 kN in Eq. 37, the required spacing for 12mm diameter stirrups s, is obtained as 94.6mm. Therefore, as shown in Fig. 3(c), a spacing of 90mm is selected.

Next, the torsional moment capacities in Modes 1, 2 and 3 of the section shown in Fig. 3(c), are computed and presented in Table 6. As can be seen, the theoretical torsional moment capacity of the section is 89.53 kN-m, and Mode 2 failure is predicted.

Example 3

Design a rectangular section if T_u = 97.75 kN-m, M_u = 324.0 kN-m and V_u = 204.0 kN.

Table 2 shows the design T/M ratio (= 0.32) to be very close to the transitional ψ of the reference section as indicated by the closeness of the k_1 and k_2 values. This closeness, however, did not pose any special problem in the solution. Since k_1 was slightly smaller, it was used in estimating M'_{01} and Mode 1 failure was assumed.

The section shown in Fig. 3(a) is proportioned with $\alpha = 1.875$. This indicates that the interaction data of the reference section presented in Table 1 can be conveniently used in the design of sections with different values of α.

The pure shear capacity of the section shown in Fig. 3(b) is presented in Table 5. As in the case of Examples 1 and 2, the reduced shear capacity $\bar{V}_n$, was less than V_n/ϕ. Accordingly, the shear capacity was increased. Calculations indicate that 14mm diameter stirrups at 110mm as shown in Fig. 3(c) would be adequate.

Table 6 gives the theoretical torsional moment capacity T_n, of the section shown in Fig. 3(c), as 137.63 kN-m.

Example 4

Design a rectangular beam section if T_u = 102.0 kN-m, M_u = 27.0 kN-m and V_u = 127.5 kN.

The high value of ψ (= 4.0) in Table 2 indicates that Mode 3 failure may occur. However, as k_3 values are not available, a section will be initially proportioned on the basis of the pure flexural moment M'_{01} estimated by using k_1 or k_2 value from Table 1. The section will then be checked for Mode 3 failure. Since Table 1 does not contain any value for $\rho = 0.012$, the required value of k_2 is obtained by interpolation. The k_2 and the estimated flexural moment M'_{01} are presented in Table 2. A section proportioned for M'_{01} are presented in Table 2. A section proportioned for M'_{01} is shown in Fig. 3(a) and its details are given in Table 3.

Design procedures similar to those used in the previous examples are then followed up to the checking of the shear capacity. The results of the various computations are presented in Tables 4 and 5. Figure 3(b) shows the section with stirrup spacing decided on the basis of Mode 2 behavior. The calculations starting with the checking of the shear capacity are presented here.

Substituting V_{01} from Table 5, in Eq. 35, $\bar{V}_n$ = 178.2 kN > V_u/ϕ (= 150.0 kN). The shear capacity is, thus, adequate although $\lambda_{01} > \lambda$ (= 0.20m). This is because, as indicated in Fig. 1(b), Eq. 35 does not control the shear capacity at high T/V ratios.

In view of the high T/M ratio, the torsional moment capacity in Mode 3 of the section shown in Fig. 3(b) is checked next. Substituting the values of K_1, Δ' and R_3 in Eq. 27, k_3 is obtained as 0.12871. Therefore, $T_3 = k_3 M_{01} = 0.12871 \times 588.57 = 75.75$ kN-m < T_u/ϕ (= 120.0 kN-m). The torsional moment capacity in Mode 3 is, therefore, to be increased at least to 120.0 kN-m. Setting $T_3 = T_u/\phi$ in Eq. 34, $R_3 = 0.21297$.

The required flexural moment capacity in negative bending is, thus, $M_{03} = R_3 M_{01} = 0.21297 \times 588.57 = 125.35$ kN-m. The corresponding required $\rho' = 0.0024$. Therefore, the required steel area at the top, $A'_s = \rho' b d_3 = 0.0024 \times 400 \times 692 = 665\text{mm}^2$. Two 22mm diameter bars having 760mm^2 area are provided as shown in Fig. 3(c).

The torsional moment capacities of this section in Modes 1, 2 and 3 are presented in Table 6. As can be seen, the section has a nominal torsional moment capacity T_n, of 126.80 kN-m and a Mode 3 failure is indicated. It may be mentioned here that the top steel area can be increased further to enhance the capacity in Mode 3 and thereby eliminate the possibility of failure in this mode.

Comparison with ACI Code Equations

The sections designed in Examples 1, 2, 3 and 4 as shown in Fig. 3(c) were investigated by using the ACI code torsion equations for comparison purpose. It was found necessary to modify the designs slightly, to satisfy the code spacing requirements, by introducing longitudinal bars at middepth and also, in some cases, by changing the hanger size. The modified sections are shown in Fig. 4. The figure also shows the torsional moment capacities of the sections as obtained by using the ACI code torsion equations, as well as those given by the interactive strength equations, along with the corresponding failure modes. As can be seen, except at a very low value of T_u, *i.e.*, in Example 1, the torsional moment capacity is consistently underestimated by the ACI code equations.

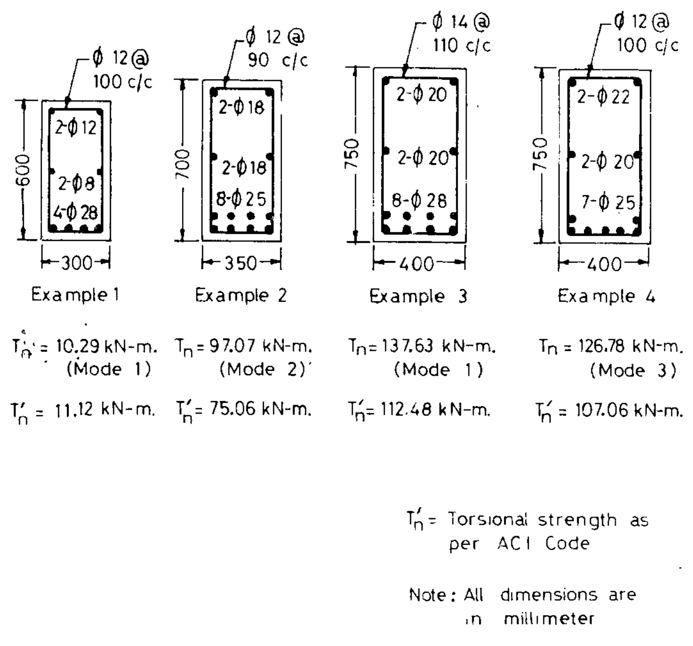


FIG. 4. Comparison with torsional strengths given by ACI code equations.

Conclusions

The interactive strength equations developed by Hasnat and Akhtaruzzaman[16] have been suitably adapted for the design of rectangular beams subjected to any combination of torsion, bending, and shear.

The design procedure basically consists of proportioning a section for a required pure flexural moment capacity. The section is then designed for torsion using the strength equations in their transformed forms.

The examples presented herein show that interaction data of a reference section are useful in estimating the required pure flexural moment capacity. The interaction data based only on Mode 1 and Mode 2 considerations are found satisfactory in most cases. However, when a possibility of Mode 3 failure exists due to high ψ and low ρ', the design is to be checked and modified accordingly if required.

It is also necessary to modify the design when M/V ratio (λ) is less than or around λ_{01} due to an overestimation of its shear strength by the basic strength equations.

Compared to the strength equations, the ACI code equations generally underestimate the torsional strength of beams subjected to torsion, bending and shear.

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Notation

The following symbols are used in this paper :

A_s, A_s'	=	area of bottom and top longitudinal steel, mm^2;
A_w	=	area of one leg of stirrup, mm^2;
b	=	breadth of beam, mm;
d, d_2, d_3	=	effective depth in positive, lateral and negative bending, respectively, mm;
f_c'	=	cylinder compressive strength, MPa;
f_{wy}	=	yield strength of web steel, MPa;
f_y	=	yield strength of longitudinal steel, MPa;
h	=	overall depth of beam, mm;
k_1, k_2, k_3	=	ratio between T_1, T_2, T_3 and M_{01};
M	=	bending moment, kN-m;
M_{01}, M_{02}, M_{03}	=	pure flexural strength in positive, lateral and negative bending respectively, kN-m;
M_{01}'	=	estimated pure flexural strength in positive bending, kN-m;
M_u	=	factored bending moment, kN-m;
r	=	$\dfrac{A_w f_{wy}}{s} \; \dfrac{0.9\, x_1\, y_1}{M_{01}}$
s	=	spacing of stirrups, mm;
T	=	torsional moment, kN-m;
T_1, T_2, T_3	=	torsional strength in Mode 1, Mode 2 and Mode 3 failure, respectively, kN-m;
T_n	=	nominal torsional moment capacity, kN-m;
T_u	=	factored torsional moment, kN-m;
V	=	shear force, kN;
V_{01}	=	pure shear capacity, kN;
V_c	=	nominal shear strength provided by concrete, kN;
V_n	=	nominal shear strength, kN;
$\bar{V}_n$	=	reduced nominal shear strength, kN;
V_s	=	nominal shear strength provided by web reinforcement, kN;
V_u	=	factored shear force, kN;
x	=	shorter overall dimension of rectangular cross-section, mm;
x_1	=	shorter center-to-center dimension of closed stirrup, mm;
y	=	longer overall dimension of rectangular cross-section, mm;
y_1	=	longer center-to-center dimension of closed stirrup, mm;
α	=	h/b;
Δ	=	$1 + \mu/\psi\lambda$;
Δ'	=	$\mu/\psi\lambda - 1$;
δ	=	$Vb/2T$;
λ	=	$M/V, m$;
λ_{01}	=	$M_{01}/V_{01}, m$;
μ	=	$(b^2 + bh)/(2b + 4h)$, mm;
ρ	=	A_s/bd;
ρ'	=	A_s'/bd;
ρ_{max}	=	$0.75 \times$ balanced steel ratio;
ϕ	=	ACI code strength reduction factor; and
ψ	=	T/M.

تصميم العتبـــات المستطيـــلة المقـــطع الخاضعـــة لـــلَّي والانحنـــاء والقـــص

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يحتوى هذا البحث على تعديل لمجموعة من معادلات المقاومة ، يعتمد بعضها على بعض ، مبنية على نموذج الانحناء المتخالف ، بحيث تصبح مناسبة لتصميم العتبات ذات المقاطع المستطيلة الخاضعة للَّي والانحناء والقص . أما طريقة التصميم فهى طريقة التجربة والخطأ وهى مبنية على تقـدير سعـة عزم الانحناء الصافي اللازمة للمقطع المراد تصميمه . وقد استعمل لتقدير سعة عزم الانحناء الصافي ، معلومات التأثير المتبادل بين اللَّي والانحناء والقص لمقطع مرجع . وقدمت هذه المعلومات في جدول وكذلك في مخططات تبادلية لا بعـدية كما قدمت أربعة أمثلة عددية تغطي طرقاً مختلفة محتملة من الانهيار . ثم قورنت سعات عزوم اللَّي للمقاطع المصممة بتلك التي نحصل عليها بتطبيق معادلات اللَّي المعطاة في قوانين الـ ACI .