

## Research Article

# Fixed Point Results in Quasimetric Spaces

**Abdul Latif and Saleh A. Al-Mezel**

*Department of Mathematics, King Abdulaziz University, P. O. Box 80203, Jeddah 21589, Saudi Arabia*

Correspondence should be addressed to Abdul Latif, latifmath@yahoo.com

Received 21 August 2010; Accepted 5 October 2010

Academic Editor: Qamrul Hasan Ansari

Copyright © 2011 A. Latif and S. A. Al-Mezel. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In the setting of quasimetric spaces, we prove some new results on the existence of fixed points for contractive type maps with respect to  $Q$ -function. Our results either improve or generalize many known results in the literature.

## 1. Introduction and Preliminaries

Let  $X$  be a metric space with metric  $d$ . We use  $S(X)$  to denote the collection of all nonempty subsets of  $X$ ,  $Cl(X)$  for the collection of all nonempty closed subsets of  $X$ ,  $CB(X)$  for the collection of all nonempty closed bounded subsets of  $X$ , and  $H$  for the Hausdorff metric on  $CB(X)$ , that is,

$$H(A, B) = \max \left\{ \sup_{a \in A} d(a, B), \sup_{b \in B} d(b, A) \right\}, \quad A, B \in CB(X), \quad (1.1)$$

where  $d(a, B) = \inf\{d(a, b) : b \in B\}$  is the distance from the point  $a$  to the subset  $B$ .

For a multivalued map  $T : X \rightarrow CB(X)$ , we say

(a)  $T$  is *contraction* [1] if there exists a constant  $\lambda \in (0, 1)$ , such that for all  $x, y \in X$ ,

$$H(T(x), T(y)) \leq \lambda d(x, y), \quad (1.2)$$

(b)  $T$  is *weakly contractive* [2] if there exist constants  $h, b \in (0, 1)$ ,  $h < b$ , such that for any  $x \in X$ , there is  $y \in I_b^x$  satisfying

$$d(y, T(y)) \leq hd(x, y), \quad (1.3)$$

where  $I_b^x = \{y \in T(x) : bd(x, y) \leq d(x, T(x))\}$ .